Depending on Types

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Is Haskell a dependently-typed language?

YES*

The Story of Dependently-typed Haskell

- *The Present*: Show how type system extensions work together to make GHC a dependently-typed language*

- *The Past*: Put those extensions in context, and talk about how they compare to dependent type theory

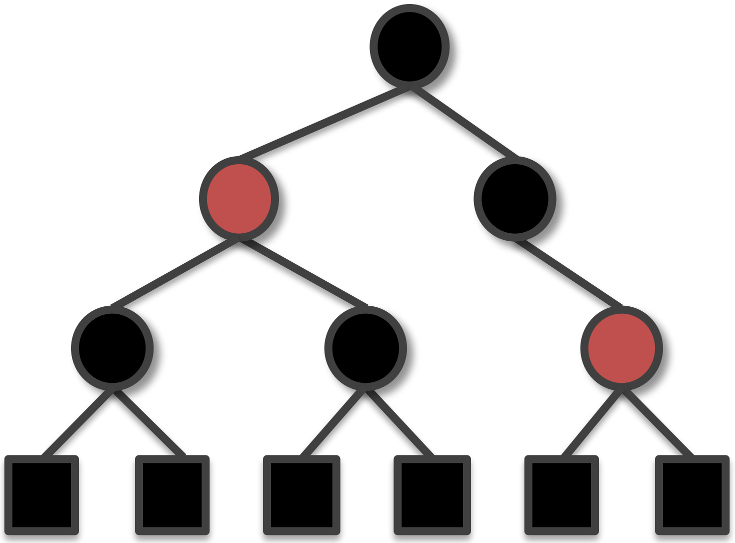
- *The Future*: Show where GHC is going

*we cannot port *every* Agda/Coq/Idris program to GHC (yet), but what we can do is impressive

Example: Red-black Trees

Running example of a data structure with application-specific invariants

- *Root is black*
- *Leaves are black*
- *Red nodes have black children*
- *From each node, every path to a leaf has the same number of black nodes*



All code available at

http://www.github.com/sweirich/dth/depending-on-types

Insertion [Okasaki, 1993]

```
data Color = R | B
data Tree  = E | T Color Tree A Tree


insert :: Tree -> A -> Tree
insert s x =          ins s
   where ins E = T R E x E
         ins s@(T color a y b)
             | x < y     =          T color (ins a) y b
             | x > y     =          T color a y (ins b)
             | otherwise = s
```

Fix the element type to be A for this talk

Temporarily suspend invariant:
Result of ins may create a red root or a red node with a red child.

Insertion [Okasaki, 1993]

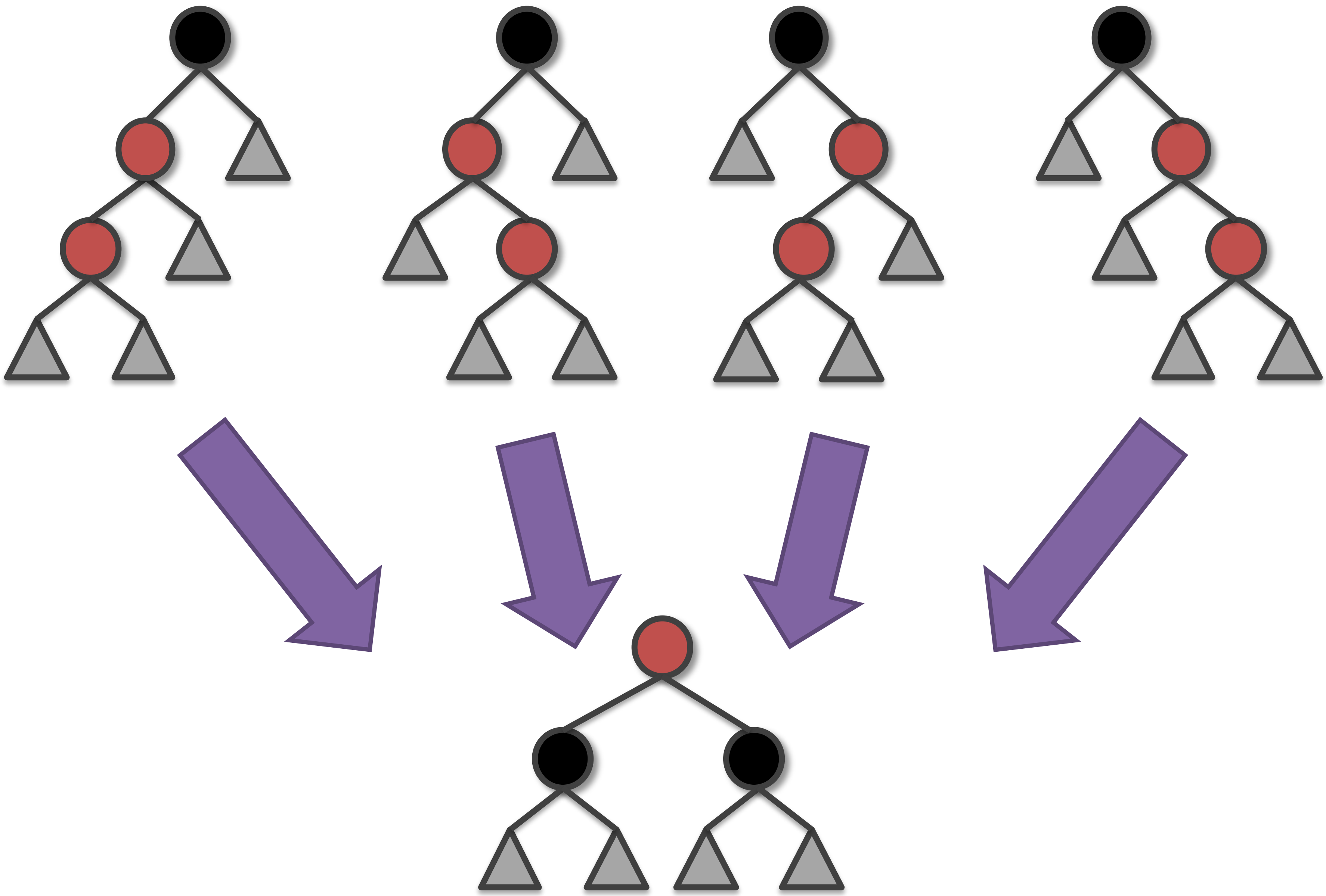
```
data Color = R | B
data Tree  = E | T Color Tree A Tree


insert :: Tree -> A -> Tree
insert s x = blacken (ins s)
   where ins E = T R E x E
         ins s@(T color a y b)
             | x < y     = balance (T color (ins a) y b)
             | x > y     = balance (T color a y (ins b))
             | otherwise = s
         blacken (T _ a x b) = T B a x b
```

Fix the element type to be A for this talk

Temporarily suspend invariant: Result of ins may create a red root or a red node with a red child.

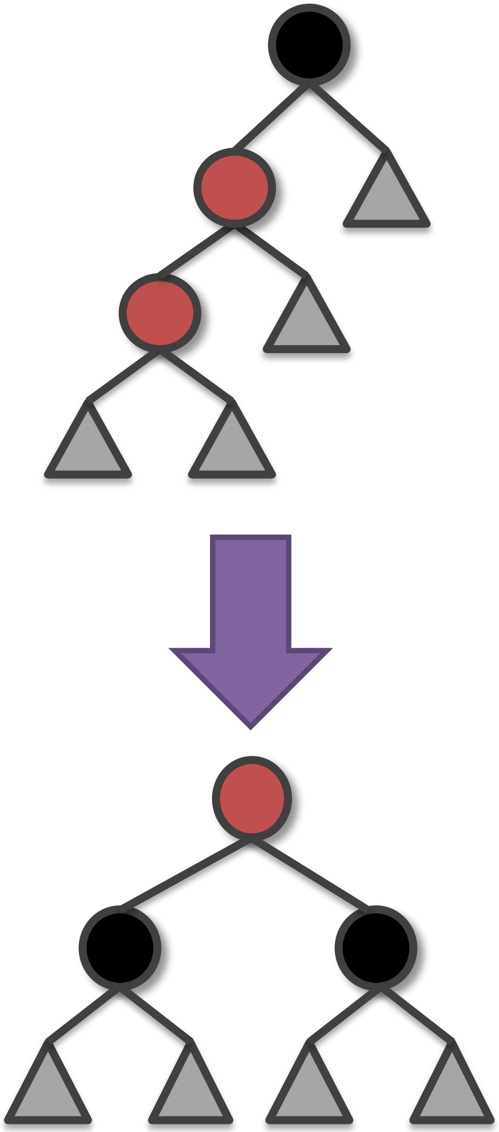
Two fixes:
- blacken if root is red at the end
- rebalance two internal reds

balance



balance [Okasaki, 1993]

```
balance :: Tree -> Tree
balance (T B (T R (T R a x b) y c) z d) =
         T R (T B a x b) y (T B c z d)
balance (T B (T R a x (T R b y c)) z d) =
         T R (T B a x b) y (T B c z d)
balance (T B a x (T R (T R b y c) z d)) =
         T R (T B a x b) y (T B c z d)
balance (T B a x (T R b y (T R c z d))) =
         T R (T B a x b) y (T B c z d)
balance (T color a x b) = T color a x b
```



How do we know insert preserves Red-black tree invariants?

Do it with types

```
insert :: RBT -> A -> RBT
```

Red-black Trees in Agda [Licata]

```
data ℕ : Set where
  Zero : ℕ
  Suc  : ℕ → ℕ

data Color : Set where
  R : Color
  B : Color

data Tree : Color → ℕ → Set where
  E  : Tree B Zero
  TR : {n : ℕ} → Tree B n → A → Tree B n → Tree R n
  TB : {n : ℕ} {c₁ c₂ : Color} →
         Tree c₁ n → A → Tree c₂ n → Tree B (Suc n)
```

Star
like

Arguments of indexed datatypes vary by data constructor.

Indexed datatype

Data constructors have dependent types. The types of later arguments depend on the values of earlier arguments.

Agda doesn't distinguish between types and terms. Curly braces indicate inferred arguments.

Enforcement with types, continued

RBT: Top-level type for red-black trees

Hides the black height and forces the root to be black

```
data RBT : Set where
  Root  : {n : ℕ} → Tree B n → RBT


insert : RBT → A → RBT
insert (Root t) x = …
```

Agda

Red-black Trees in GHC

```
data Tree : Color → ℕ → Set where
  E  : Tree B Zero
  TR : {n : ℕ} → Tree B n → A → Tree B n → Tree R n
  TB : {n : ℕ} {c₁ c₂ : Color} →
           Tree c₁ n → A → Tree c₂ n → Tree B (Suc n)
```

Agda

```
data Tree :: Color -> Nat -> * where
   E  :: Tree B Zero
   TR :: Tree B n -> A -> Tree B n -> Tree R n
   TB :: Tree c1 n -> A -> Tree c2 n -> Tree B (Suc n)
```

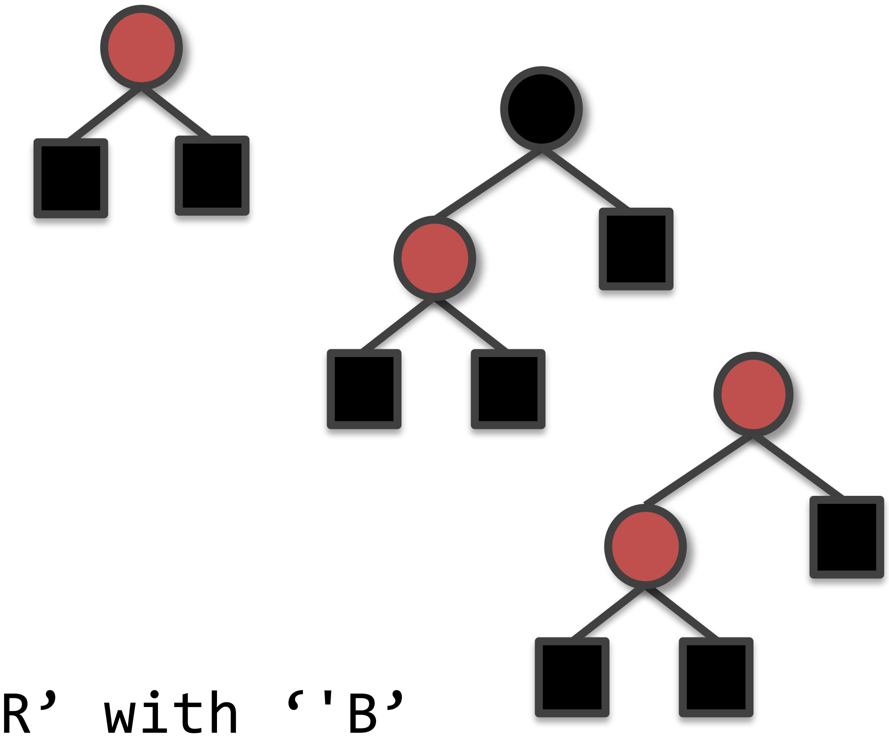
Haskell

GADTs - datatype arguments may vary by constructor

Datatype promotion – data constructors may be used in types (which are naturally dependent)

Static enforcement

```
ghci> let t1 = TR E a1 E
ghci> :type t1
t1 :: Tree 'R 'Zero
ghci> let t2 = TB t1 a2 E
ghci> :type t2
t2 :: Tree 'B ('Suc 'Zero)
ghci> let t3 = TR t1 a2 E
<interactive>:38:13:
    Couldn't match type ‘'R’ with ‘'B’
    Expected type: Tree 'B 'Zero
      Actual type: Tree 'R 'Zero
    In the first argument of ‘TR’, namely ‘t1’
    In the expression: TR t1 A2 E
```



Enforcement with types, continued

RBT: Top-level type for red-black trees

Hides the black height and forces the root to be black

```
data RBT : Set where
  Root  : {n : ℕ} → Tree B n → RBT


insert : RBT → A → RBT
insert (Root t) x = …
```

Agda

Haskell

```
data RBT :: * where
  Root :: Tree B n -> RBT


insert :: RBT -> A -> RBT
insert (Root t) x = …
```

Agda and Haskell look similar

- Tree reversal swaps the order of elements in the tree
- Indexed types prove that black height is preserved and root color unchanged

```
rev : {n : ℕ} {c : Color} → Tree c n → Tree c n
rev E = E
rev (TR a x b) = TR (rev b) x (rev a)      -- a, b : Tree B n
rev (TB a x b) = TB (rev b) x (rev a)
```

Agda

```
rev :: Tree c n -> Tree c n
rev E = E
rev (TR a x b) = TR (rev b) x (rev a)
rev (TB a x b) = TB (rev b) x (rev a)
```

Haskell

For the application of TR to type check, we must know that (rev b) and (rev a) are black trees of equal height.

How are Agda and Haskell different?

Haskell distinguishes types from terms

Agda does not

Types are special in Haskell:

1. Type arguments are always inferred (HM type inference)
2. Only types can be used as indices to GADTs
3. Types are always erased before run-time

GADTs: *Type* indices only

- Both Agda and GHC support indexed datatypes, but GHC syntactically requires indices to be *types*
- Datatype promotion automatically creates new data*kinds* from data*types*

```
data Color :: * where  -- Color is both a type and a kind
  R :: Color           -- R and B can appear in both
  B :: Color           -- expressions and types

data Tree :: Color -> Nat -> * where
  E  :: Tree B Zero
  TR :: Tree B n -> A -> Tree B n -> Tree R n
  TB :: Tree c1 n -> A -> Tree c2 n -> Tree B (Suc n)
```

Types are erased

RBT: Top-level type for red-black trees

Hides the black height and forces the root to be black

```
data RBT : Set where
  Root   : {n : ℕ} → Tree B n → RBT


bh : RBT -> ℕ
bh (Root {n} t) = n
```

Agda

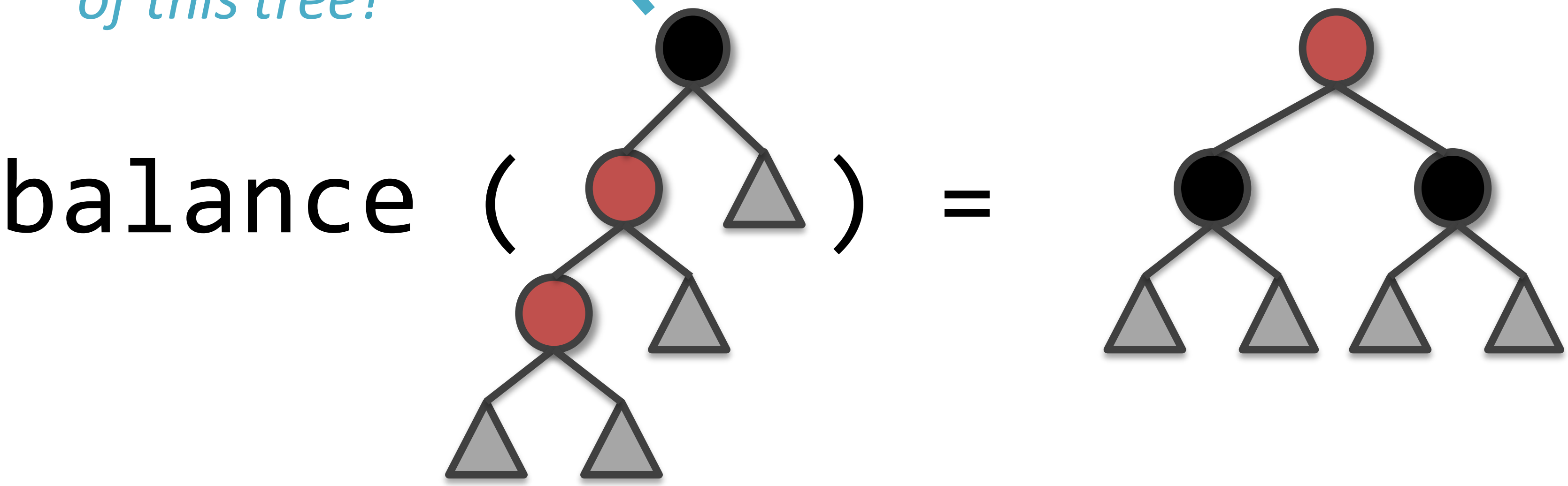
Haskell

```
data RBT :: * where
  Root :: Tree B n -> RBT


bh :: RBT -> Nat
bh (Root t) = ???         -- No runtime access to black height
```

Insertion

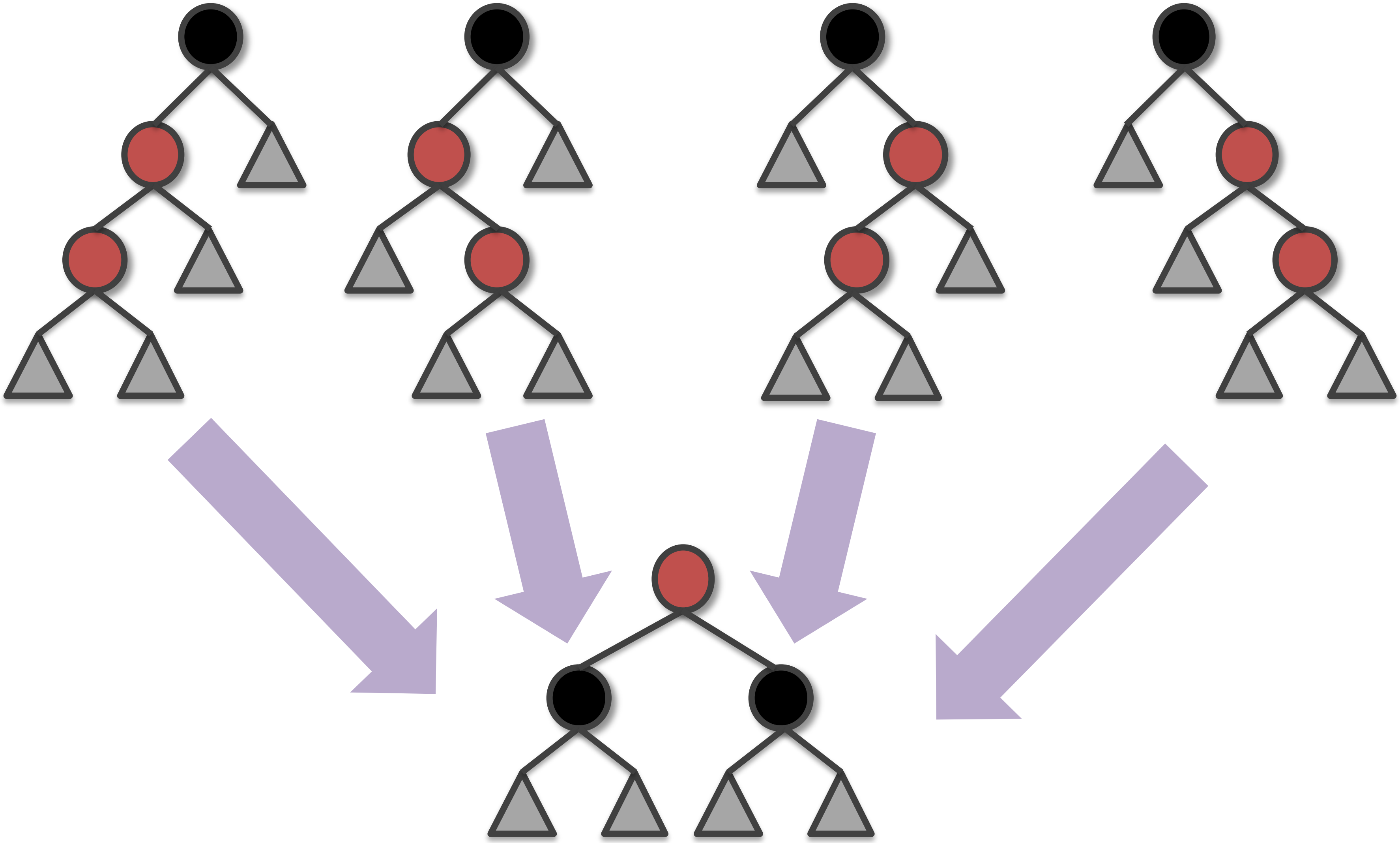
How do we temporarily suspend the invariants during insertion?

What is the type of this tree?



balance ( ) =

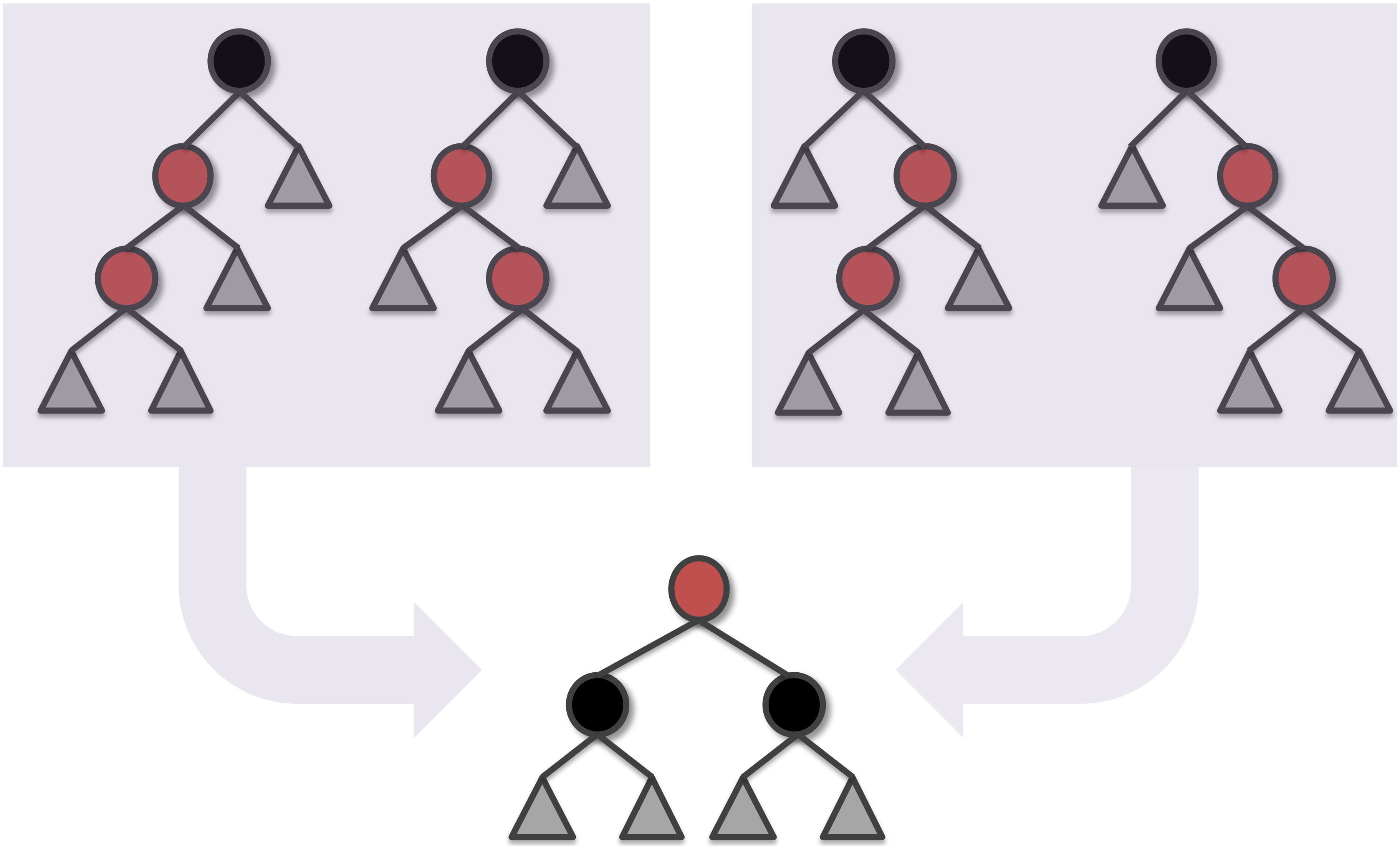
balance



`balance (TB (ins a) y b)`

`balance (TB a y (ins b))`

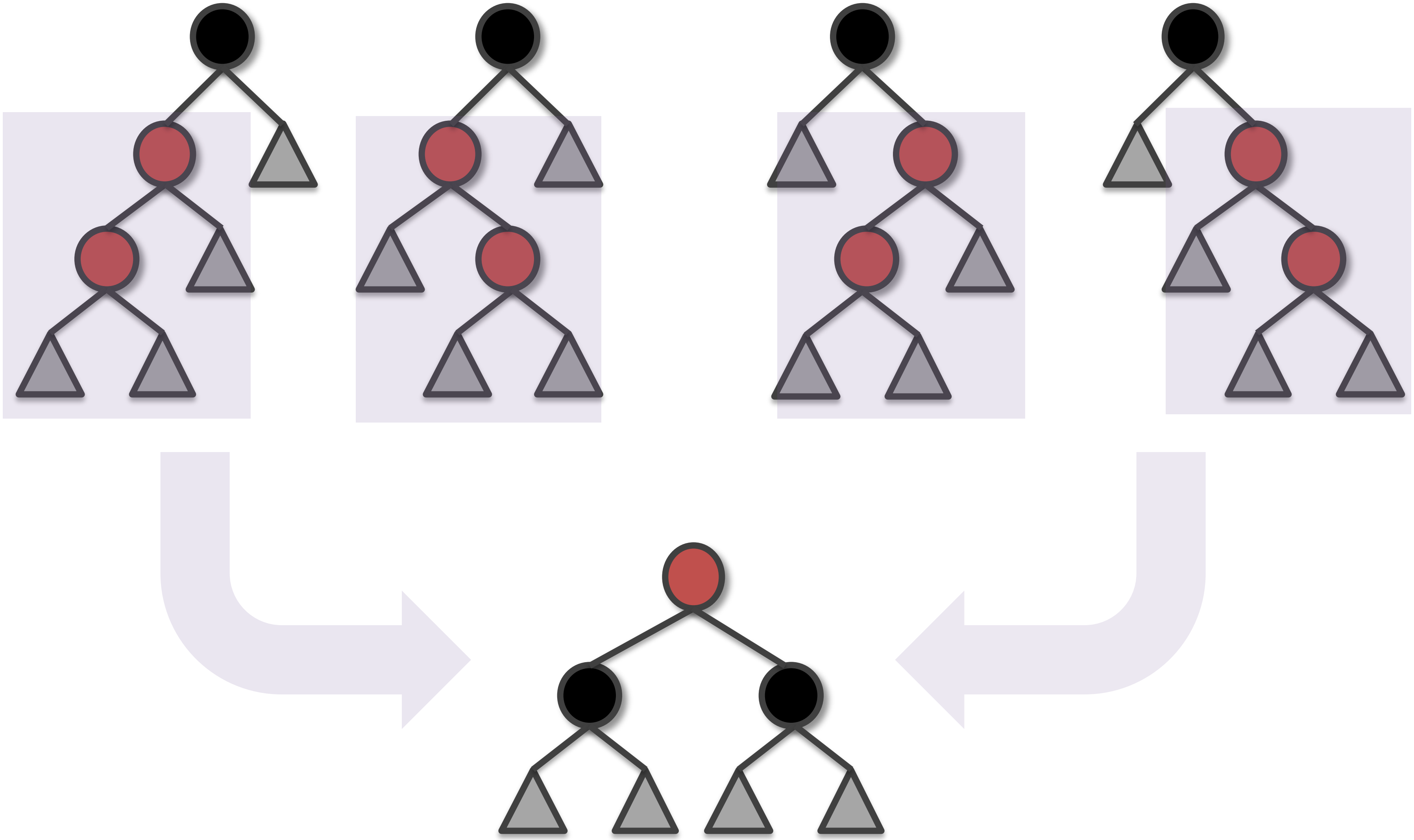
Split balance into two functions



```
balanceL (TB (ins a) y b)
```

```
balanceR (TB a y (ins b))
```

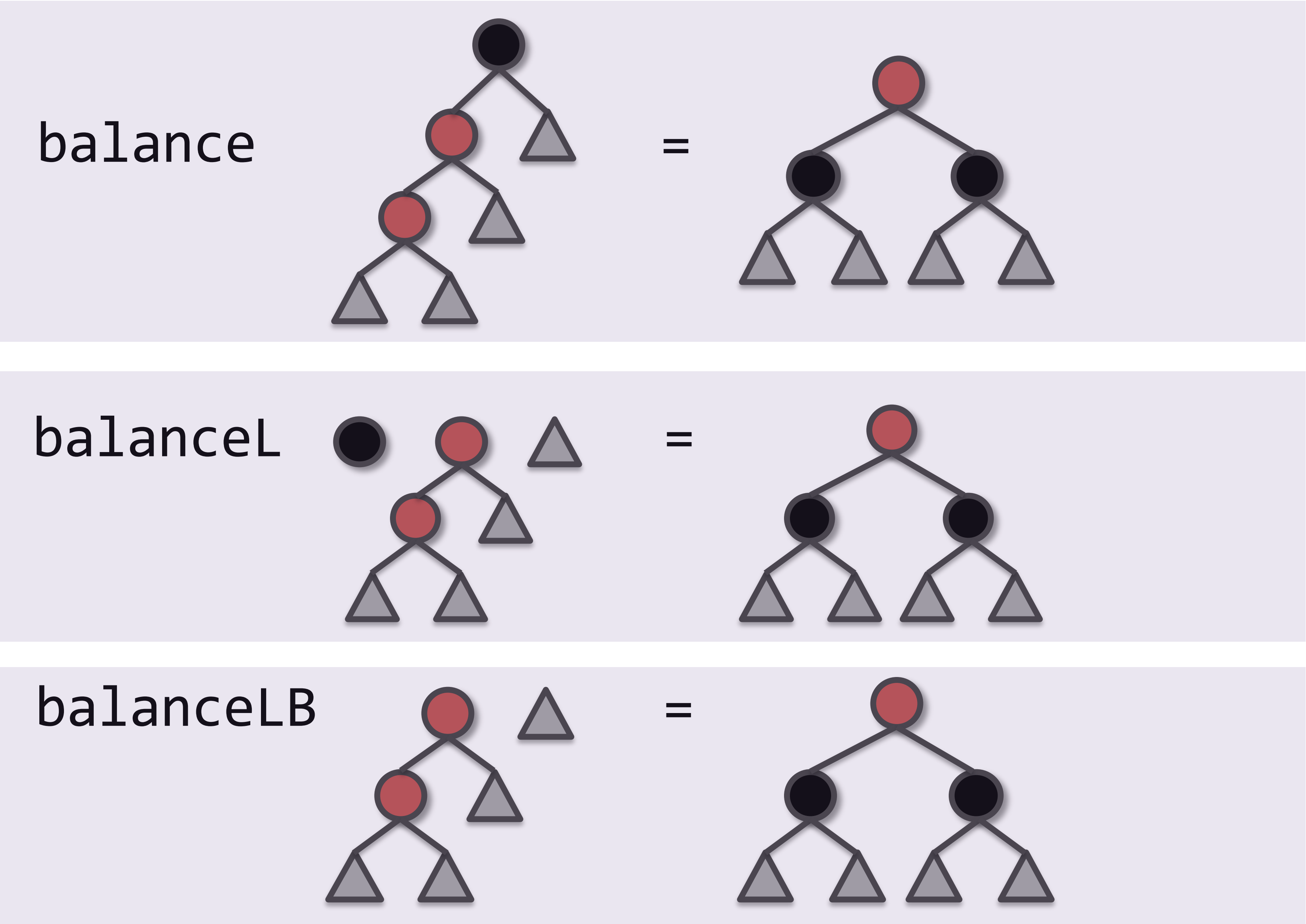
Remove top-level node



```
balanceLB (ins a) y b
```

```
balanceRB a y (ins b)
```

Specialize to Color

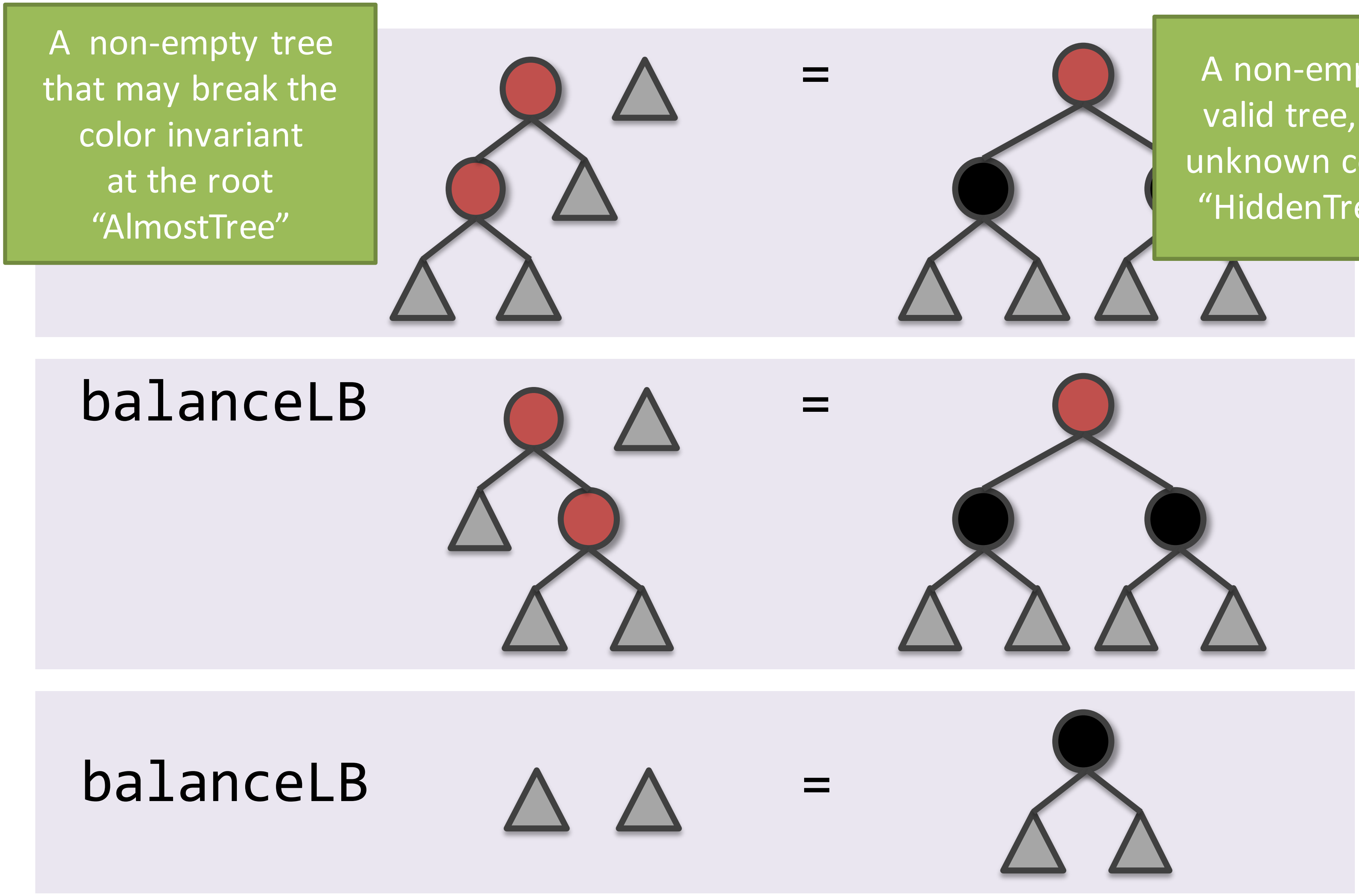


balance =

balanceL =

balanceLB =

```
balanceLB :  ??? → A → Tree c n → ???
```



A non-empty tree that may break the color invariant at the root “AlmostTree”

A non-empty valid tree, of unknown color “HiddenTree”

Programming with types (Agda)

- A non-empty valid tree, of unknown color

```
  data HiddenTree : ℕ → Set where
     HR : {m : ℕ} → Tree R m       → HiddenTree m
     HB : {m : ℕ} → Tree B (Suc m) → HiddenTree (Suc m)
```

- A non-empty tree that may break the invariant at the root

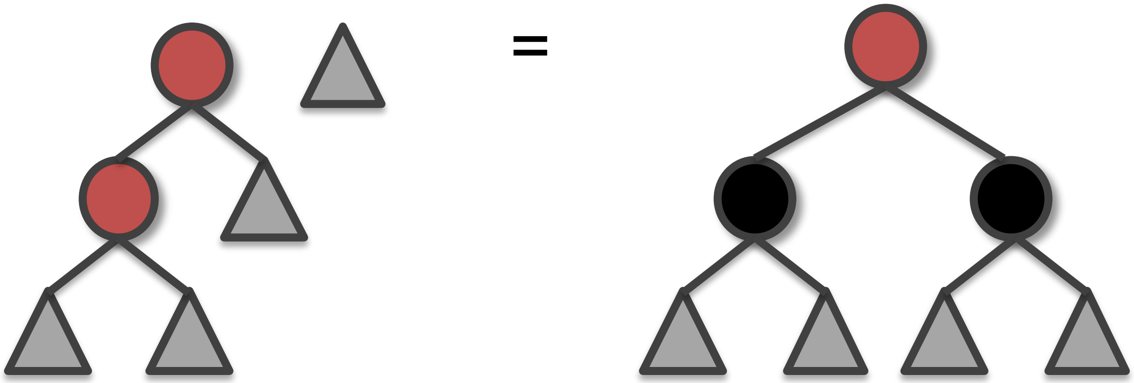
```
incr : Color → ℕ → ℕ
incr B = Suc
incr R = id


data AlmostTree : ℕ → Set where
  AT : {n : ℕ}{c₁ c₂ : Color} → (c : Color) →
       Tree c₁ n → A → Tree c₂ n → AlmostTree (incr c n)
```

Use a function to calculate the black height from the color

Agda

```
balanceLB : {n : ℕ}{c : Color} →
      AlmostTree n → A → Tree c n → HiddenTree (Suc n)
balanceLB (AT R (TR a x b) y c) z d =
  HR (TR (TB a x b) y (TB c z d))
balanceLB (AT R a x (TR b y c)) z d =
  HR (TR (TB a x b) y (TB c z d))
balanceLB (AT B a x b) y r = HB (TB (TB a x b) y r)
balanceLB (AT R E x E) y r = HB (TB (TR E x E) y r)
balanceLB (AT R (TB a w b) x (TB c y d)) z e =
  HB (TB (TR (TB a w b) x (TB c y d)) z e)
```

balanceLB



GHC version of AlmostTree

```
type family Incr (c :: Color) (n :: Nat) :: Nat where
   Incr R n = n
   Incr B n = Suc n

data Sing :: Color -> * where
   SR :: Sing R
   SB :: Sing B

data AlmostTree :: Nat -> * where
   AT :: Sing c -> Tree c1 n -> A -> Tree c2 n ->
        AlmostTree (Incr c n)
```

Type family

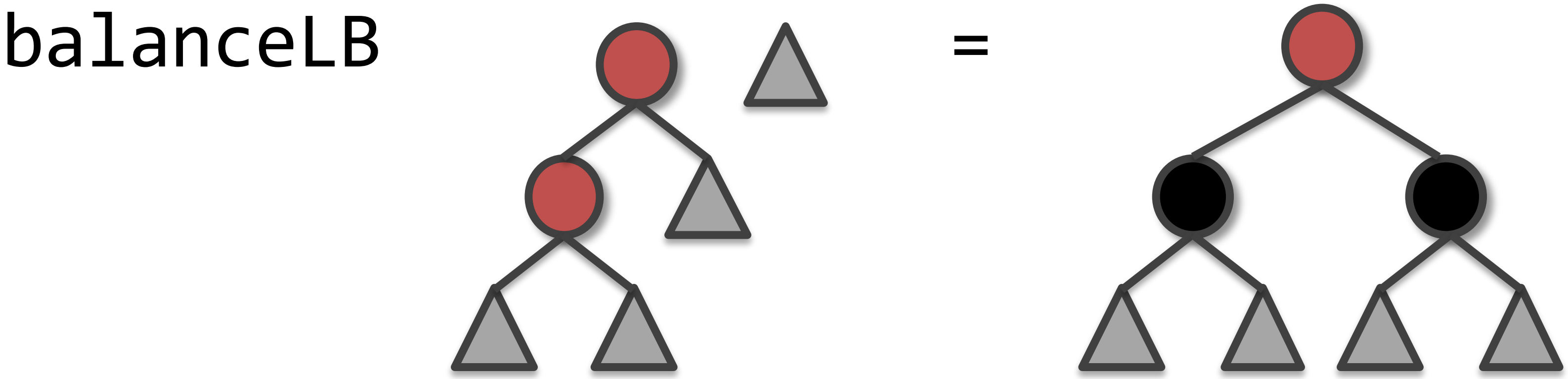
Singleton type

Type-term separation:
Singleton types provides runtime access to the color of the node in GHC.

See also: singletons library

Agda

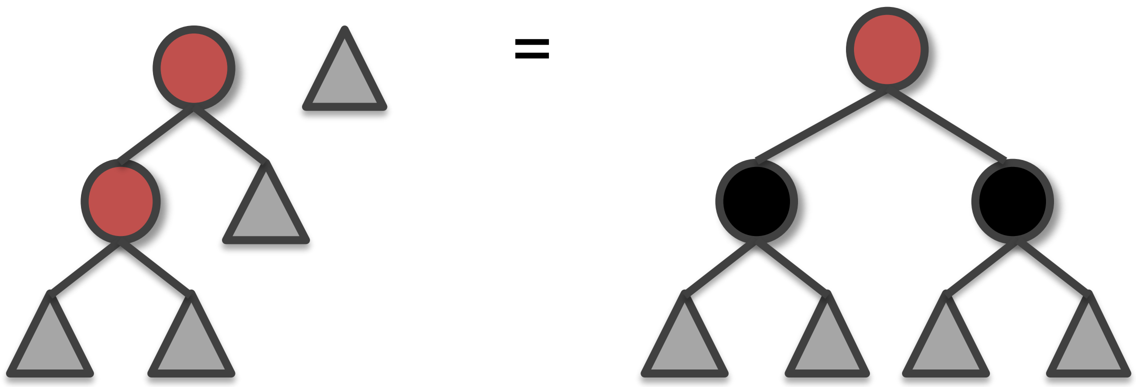
```
balanceLB : {n : ℕ}{c : Color} →
      AlmostTree n → A → Tree c n → HiddenTree (Suc n)
balanceLB (AT R (TR a x b) y c) z d =
  HR (TR (TB a x b) y (TB c z d))
balanceLB (AT R a x (TR b y c)) z d =
  HR (TR (TB a x b) y (TB c z d))
balanceLB (AT B a x b) y r = HB (TB (TB a x b) y r)
balanceLB (AT R E x E) y r = HB (TB (TR E x E) y r)
balanceLB (AT R (TB a w b) x (TB c y d)) z e =
  HB (TB (TR (TB a w b) x (TB c y d)) z e)
```



Haskell

```
balanceLB ::
      AlmostTree n -> A -> Tree c n -> HiddenTree (Suc n)
balanceLB (AT SR (TR a x b) y c) z d =
   HR (TR (TB a x b) y (TB c z d))
balanceLB (AT SR a x (TR b y c)) z d =
   HR (TR (TB a x b) y (TB c z d))
balanceLB (AT SB a x b) y r = HB (TB (TB a x b) y r)
balanceLB (AT SR E x E) y r = HB (TB (TR E x E) y r)
balanceLB (AT SR (TB a w b) x (TB c y d)) z e =
   HB (TB (TR (TB a w b) x (TB c y d)) z e)
```

balanceLB



Implementation of insert

- The Haskell version of insert is in lock-step with Agda version!
- But, are they the same? Not quite…

  Agda:

```
insert : RBT → A → RBT
```

given a (valid) red-black tree and an element,
insert ***will*** *produce a valid red-black tree*

Haskell:

```
insert :: RBT -> A -> RBT
```

given a (valid) red-black tree and an element,
if *insert produces a red-black tree,* ***then*** *it will be valid*

Difference: Totality

Adga requires all functions to be proved total

Haskell does not

- On one hand, Agda provide stronger guarantees about execution.
- On the other hand, totality checking is inescapable. Sometimes *not* reasoning about totality simplifies dependently-typed programming.

Not proving things is simpler

- Okasaki's version of insert (simply typed): 12 lines of code
- Haskell version translated from Agda: 49 lines of code
  - includes type defs & signatures
  - precise return types for balance functions

```
balanceLB :: AlmostTree n -> A -> Tree c n -> HiddenTree (Suc n)
balanceLR :: HiddenTree n -> A -> Tree c n -> AlmostTree n
```

- Haskell version from scratch (see git repo) : 32 lines of code
  - includes type defs & signatures
  - more similar to Okasaki's code
  - less precise return type for balance functions

```
balanceL :: Sing c ->
          AlmostTree n -> A -> Tree c n -> AlmostTree (Incr c n)
```

What’s next for GHC

Extensions in Progress

- Datatype promotion only works once
  - Cannot use dependently-typed programming at the type level
  - Some Agda programs have no GHC equivalent
  - Theory for GHC Core [Weirich,Hsu,Eisenberg; ICFP 2013]
  - Current status: Richard Eisenberg's implementation available https://github.com/goldfirere/ghc
  - Merging in to HEAD for GHC 8.0

- GHC should have a real dependent type
  - Plan: Identify a shared subset of types and terms, introduce a new quantifier (Π) over that subset
  - Adam Gundry's dissertation provides a road map
  - Richard plans to implement

Why dependently-typed Haskell?

TDD

Type-Driven Development

The Agda Experience

On 2012-01-11 03:36, Jonathan Leivent wrote on the Agda mailing list:

> Attached is an Agda implementation of Red Black trees [..]
> The dependent types show that the trees have the usual
> red-black level and color invariants, are sorted, and
> contain the right multiset of elements following each function. [..]

> However, one interesting thing is that I didn't previously know or
> refer to any existing red black tree implementation of delete - I
> just allowed the combination of the Agda type checker and
> the exacting dependent type signatures to do their thing [..]
> **making me feel more like a facilitator than a programmer**.

What else do we need?

- Totality checking for GHC
  - Pattern match exhaustiveness and termination
  - Language should give programmers the choice [Trellys]

- Type inference beyond Hindley-Milner
  - Unsaturated type families
  - First-class polymorphism
  - Special purpose constraint solvers [Iavor Diatchki]
  - Programmable error messages

- Programming support
  - Automatic case splitting
  - Automatic code completion and code synthesis

Conclusion

Haskell programmers can use dependent types*

… and we're actively working on the *

… but it is exciting to think about how *dependent*-type structure can help design programs

Thanks to: Simon Peyton Jones, Dimitrios Vytiniotis, Richard Eisenberg, Brent Yorgey, Geoffrey Washburn, Conor McBride, Adam Gundry, Iavor Diatchki, Julien Cretin, José Pedro Magalhães, David Darais, Dan Licata, Chris Okasaki, Matt Might, NSF

http://www.github.com/sweirich/dth

Where do these features come from?

Datatype promotion

- Makes the type-term separation less brutal
  - Automatically allows data structures to be used in types
  - Includes kind-polymorphism (for promoting lists…)
  - Limitation: GADTs can't be promoted (*more on that later)

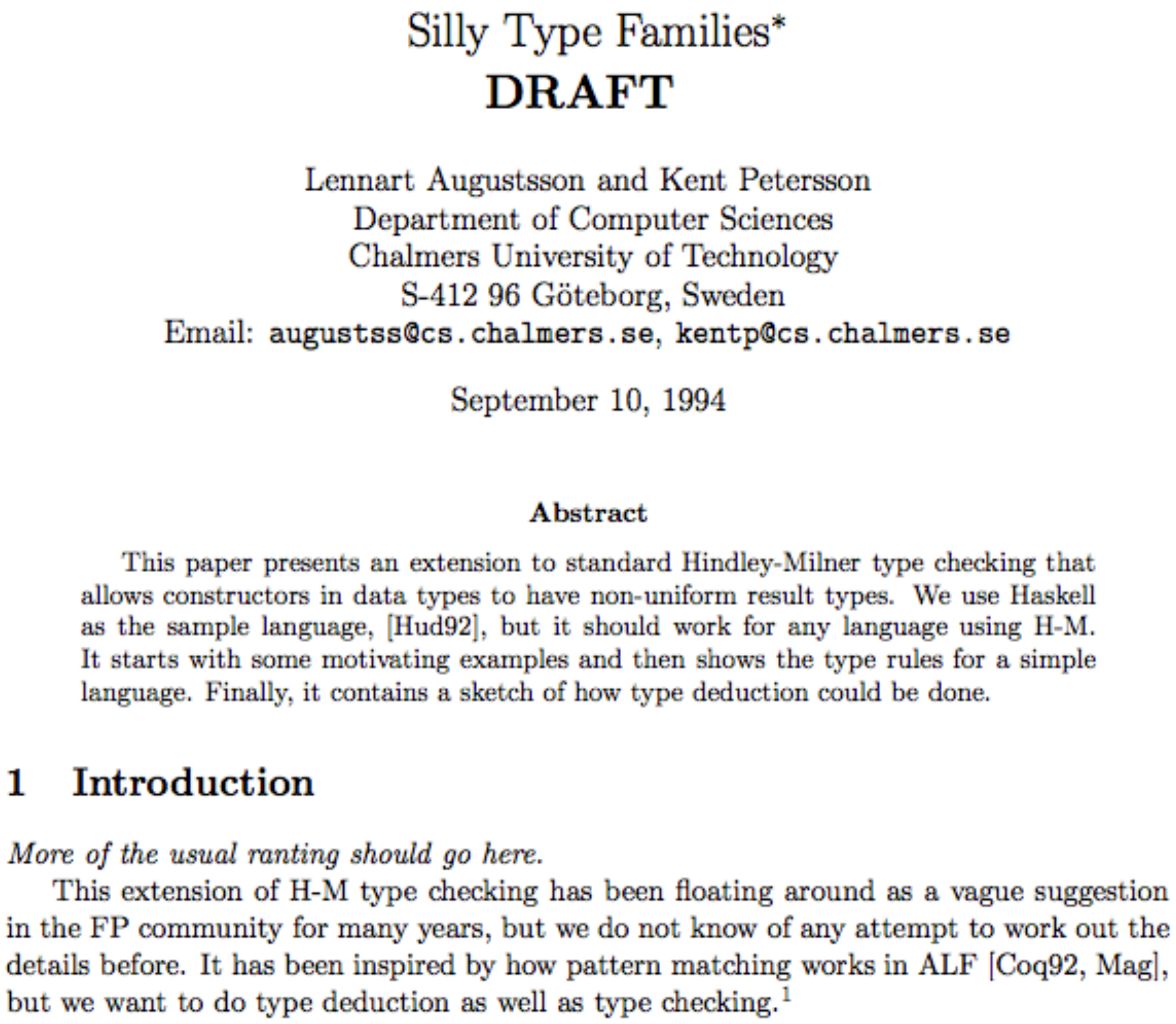
- Recent extension

  [Yorgey, Weirich, Cretin, Peyton Jones, Vytiniotis, Magalhães; TLDI 2012]
  - Inspired by Strathclyde Haskell Extension (SHE) [McBride]
  - Introduced in GHC 7.4 [Feb 2012]

"It's crazy how cool the features in new GHC releases are. Other languages get patches to prevent some buffer overflow, we get patches to add an entirely new level of polymorphism." -mbetter on Reddit

GADTs

- Not so recent: Introduced in GHC 6.4 [March 2005]
- Many pre-cursors:
  - [Cheney, Hinze 2003] First-class phantom types (Haskell encoding)
  - [Xi, Chen, Chen 2003] Guarded Recursive Datatypes (ATS)
  - [Sheard, Pasalic 2004] Equality qualified types (Ωmega)
  - [Peyton Jones, Washburn, Weirich 2004] Generalized Algebraic Datatypes (Haskell primitive)
  - [Simonet, Pottier 2005] Guarded Algebraic Types (OCaml)
- Challenge: Integration with Hindley-Milner type inference
  - [Pottier, Régis-Gianis; POPL 2006]
  - [Peyton Jones, Vytiniotis, Washburn, Weirich; ICFP 2006]
  - [Sulzmann, Chakravarty, Peyton Jones; TLDI 2007]
  - [Schrijvers, Peyton Jones, Sulzmann, Vytiniotis; ICFP 2009]
- Could have been added to GHC much earlier…



Silly Type Families*
DRAFT

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September 10, 1994

Abstract

This paper presents an extension to standard Hindley-Milner type checking that allows constructors in data types to have non-uniform result types. We use Haskell as the sample language, [Hud92], but it should work for any language using H-M. It starts with some motivating examples and then shows the type rules for a simple language. Finally, it contains a sketch of how type deduction could be done.

1 Introduction

More of the usual ranting should go here.

This extension of H-M type checking has been floating around as a vague suggestion in the FP community for many years, but we do not know of any attempt to work out the details before. It has been inspired by how pattern matching works in ALF [Coq92, Mag], but we want to do type deduction as well as type checking.[1]

Singleton types

- Standard trick for languages with a type-term distinction
  [Hayashi 1991][Xi, Pfenning 1998]

```
data Sing :: Color -> * where
    SR :: Sing R    -- SR only non-⊥ inhabitant of Sing R
    SB :: Sing B
```

- Can be as expressive as a full-spectrum language
  [Monnier, Haguenauer; PLPV 2010]

```
(x : A) → B  ➔  forall (x :: A). Sing x -> B
```

- In GHC
  - Haskell library (singletons) automates translation, though limited by datatype promotion restrictions* [Eisenberg, Weirich; HS 2012]
  - Extensive use of singletons is painful* [Lindley,McBride; HS 2013]

Type families

- Motivation
  - Highly parameterized library interfaces

```
class IsList l where
   type Item l
   fromList :: [Item l] -> l
   toList :: l -> Item l
```

```
instance IsList Text where
   type Item = Char
   fromList = Text.pack
   toList = Text.unpack
```

  - Generic programming (type-indexed types)
  - Move to replace “logic programming” style of type-level computation (MPTC+FD) with “functional programming” style

- Challenge: Integration with Hindley-Milner type inference

  [Chakravarty, Keller, Peyton Jones, Marlow; POPL 2005]

  [Chakravarty, Keller, Peyton Jones; ICFP 2005]

  [Schrijvers, Peyton Jones, Chakravarty, Sulzmann; ICFP 2008]

  [Eisenberg, Vytiniotis, Peyton Jones, Weirich; POPL 2014]

Type families are not functions

- More restrictive:
  - No lambdas (must be named)
  - Application must be saturated
  - Restrictions on unification
- More expressive:
  - Can pattern match types, not just data
  - Equality testing is available for any kind

```
type family Id (a :: *) where
     Id a = a

instance Monad Id where
     …
```

```
type family F (a :: Nat) where
     F Zero = Int

f :: F a -> F a
f x = x
```

```
type family Item (a :: *) where
     Item Text = Char
     Item [a]  = a
```

```
type family Eq (a :: k) (b :: k) :: Bool where
     Eq a a = True
     Eq a b = False
```

Beyond Red-black Trees

Embedded Domain-Specific Languages
need
Embedded Domain-Specific Type Systems

Experience Reports

- *Ivory*: embedding safe-C types in Haskell's type system
  [Hickey et al; ICFP 2014]
- *HarmTrace*: a system for automatically analyzing the harmony of music sequences
  [Magalhães, de Haas; ICFP 2011]
- *Units*: tracking astrophysical units
  [Muranushi, Eisenberg; HS 2014]

Haskell (Kahrs 2001)

Datatype structure ensures color invariant

Phantom type tracks depth (not height)

```
data Suc n
data Black n     = E
  | TB (RedOrBlack (Suc n)) A (RedOrBlack (Suc n))
data RedOrBlack = C (Black n) | TR (Black n) A (Black n)
data RBT = forall n. Root (Black n A)


-- needs `phantom coercion’
inc :: Tree a n -> Tree a n’
inc = …
```

Music composition

```
-- Diatonic fifths, and their class (comments with the
   CMaj scale)
-- See http://en.wikipedia.org/wiki/Circle_progression

type family DiatV deg :: *
type instance DiatV I   = Imp -- V   -- G7  should be Dom
type instance DiatV V   = Imp -- II  -- Dm7 should be
   SDom
type instance DiatV II  = VI  -- Am7
type instance DiatV VI  = III -- Em7
type instance DiatV III = VII
  -- Bhdim7 can be explained by Dim rule
type instance DiatV VII = Imp -- IV
  -- FMaj7 should be SDom
type instance DiatV IV  = Imp -- I   -- CMaj7
```

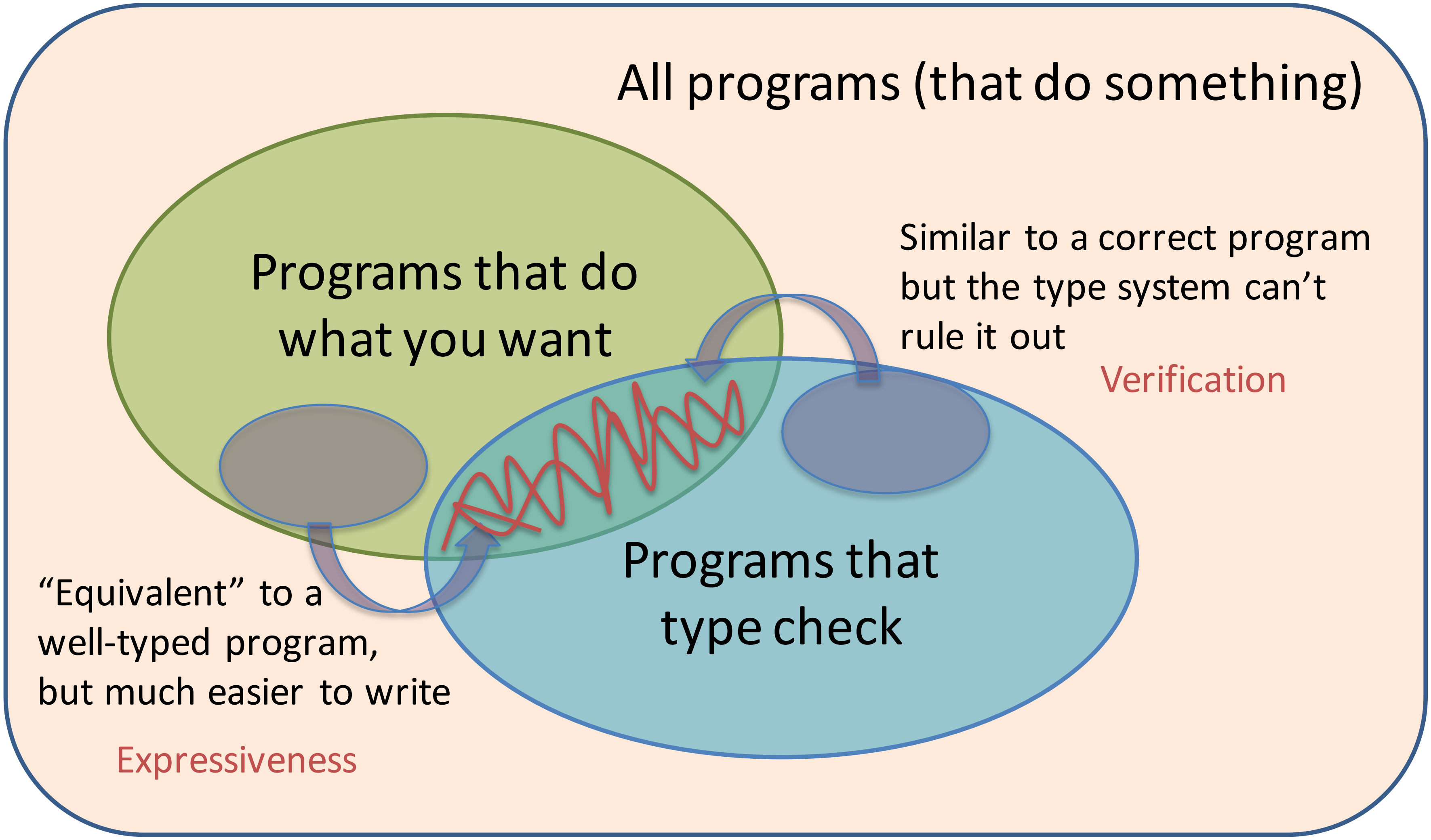
http://hackage.haskell.org/package/HarmTrace-2.2.0

Ivory

```
-- | Convert an array of four 8-bit integers into a 32-bit integer.
test2 :: Def ('[Ref s (Array 4 (Stored Uint8))]
               :-> Uint32)
test2 = proc "test2" $ \arr -> body $ do
  a <- deref (arr ! 0)
  b <- deref (arr ! 1)
  c <- deref (arr ! 2)
  d <- deref (arr ! 3)
  ret $ ((safeCast a) `iShiftL` 24) .|
         ((safeCast b) `iShiftL` 16) .|
         ((safeCast c) `iShiftL` 8)  .|
         ((safeCast d) `iShiftL` 0)
```

https://github.com/GaloisInc/ivory

Type systems Research



All programs (that do something)

Programs that do what you want

Programs that type check

Similar to a correct program but the type system can't rule it out

Verification

"Equivalent" to a well-typed program, but much easier to write

Expressiveness

Why Dependent Types?

- *Verification*: Dependent types express application-specific program invariants that are beyond the scope of existing type systems
- *Expressiveness*: Dependent types enable flexible interfaces, of particular importance to generic programming and metaprogramming.
- *Uniformity*: The same syntax and semantics is used for computations, specifications and proofs

Program verification is “just programming”

Making the type checker programmable